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ESTIMATING CURRENT TREND AND GROWTH RATES IN SEASONAL TIME SERI--ETC(U)
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ESTIMATING CURRENT TREND AND GROWTH RATES IN SEASONAL TIME SERIES

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UNIVERSITY OF WISCONSIN-MADISON
MATHEMATICS RESEARCH CENTER

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ABSTRACT

The importance of appropriate stochastic models in choosing efficient methods of statistical analysis is discussed. The fitting to data of Seasonal Autoregressive Moving Average models is described and it is shown how trend may be estimated in an appropriate class of models of this kind. The procedure is illustrated for a model fitted to a money supply series published by the Federal Reserve Board. Error limits are calculated. In a series of Appendices the properties of the adaptive coefficients which determine the trend estimates are derived.

AMS (MOS) Subject Classifications: 62M10

Key Words: Seasonal Adjustment, Trend Estimation, Time Series Models, Adaptive Estimation

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SIGNIFICANCE AND EXPLANATION

Seasonal fluctuations in time series often obscure information about trend. Thus it may be known that recruitment for the Army regularly decreases in certain months of the year and therefore that reductions in these months are not to be regarded as indicating any real change in the recruitment situation. Historically problems of this kind have been dealt with by making "seasonal adjustments" but the manner in which these adjustments were made has been somewhat arbitrary is occasionally misleading.

It is argued in this paper that the problem can best be dealt with not by seasonal adjustment at all but by direct estimation of the current trend using an appropriate stochastic model which has been fitted to the series. The method has the advantage that, provided appropriate precautions, which are discussed, are taken to ensure that the model is adequate, the estimates of trend will have efficient properties and they will employ the data in a manner most appropriate to the particular series under study.

ESTIMATING CURRENT TREND AND GROWTH RATES IN SEASONAL TIME SERIES

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1. INTRODUCTION

For decades the seasonal adjustment of time series has been widely practiced and even more widely discussed. The goal of such adjustment is presumably to facilitate the elucidation and interpretation of other systematic aspects of the series, i.e., of the trend. Thus, in one sense the title of this paper is another name for seasonal adjustment. However, our approach differs in several important respects from the Census X11 procedure (Shiskin, Young, and Musgrave, 1967) and other traditional approaches to deseasonalization. Most importantly, it is linked to building a model for the series, thereby basing whatever further analysis is desired on that model's established properties; this aspect is more fully discussed in Section 2. A further point is that our focus is on current and projected trend rather than on extracting patterns in historical series. While ex post seasonal adjustment and trend estimation are important for some purposes, the problem of prime importance to many people, including forecasters, planners and policymakers, is obtaining the best estimate of what is happening now. Finally, the procedure is determined uniquely by the model and thus avoids the elements of arbitrariness inherent in stochastic modelling approaches such as those of Box, Hillmer, and Tiao (1978) and Pierce (1978).

Section 2 of the paper discusses the importance of model-building in scientific inquiry and introduces the time series models on which the

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present investigation is based. In Section 3 these models are used to develop estimates of trend for general seasonal series. This involves an analysis of the forecast function of the modelled series, expressed as the solution of a difference equation. This solution contains a set of adaptive coefficients (adaptive to the current time period), some of which can be associated with the series' current trend (or growth rate for logged series).

Section 4 studies modelling and trend estimation for the series of demand deposits at commercial banks, a major component of the U.S. money supply. Estimated and actual growth rates are compared over a 5-year period, and their mean square error is found to correspond closely to the demand-deposit model's error estimate. Some concluding remarks comprise Section 5.

2. TIME SERIES MODELS AND MODELLING PHILOSOPHY

Seasonal adjustment, trend estimation and so forth imply a belief in regularities of some kind in the series studied. This "sequence regularity" is imperfect and therefore must be expressed in terms of probability. Sequence regularity expressed by probability is precisely what a time series model is. While a time series model implies a particular choice of procedure, a particular choice of procedure implies a particular kind of time series model. Thus, it was shown by Cleveland and Tiao (1976) that the X11 program implies (very nearly) a particular kind of time series model with particular values of its adjustable parameters. Let us call this model M(X11). Since X11 was arrived at by highly skilled people using many iterations, M(X11) is an average or compromise model for the range of series on which its organizors tested it. Consequently, X11 can be expected to do an averagely good job for series whose models are similar to M(X11) (for series which are like the ones it was tested on) and not as good job on other series. For example, if X11 were applied to a random series, it would <u>induce</u> seasonality in the series.

The above implies, of course, that models and methods are not arbitrary but can be built and applied in a series of logical steps. These steps have been set out, for example, by Box and Jenkins (1970). Basically, the argument is that a model is a transformation of data to white noise, evidenced by residuals, uncorrelated with any other known input. The model building process is an iteration which is guided at each step by the need to achieve this.

The conclusion from this argument is as follows. Since different time series have different probability structures (i.e., different models), methods of analysis such as seasonal adjustment methods, trend estimation methods, etc., should be different and should depend on the model. Similarly,

seasonal adjustment methods, if they are really relevant to the problem, must be model-based [as are those proposed by Box, Hillmer, and Tiao (1978) and Pierce (1978), and Burman (1980)], so that they adapt to the series under study.

In any statistical or scientific investigation, therefore, one's approach should consist of two stages:

1) to build a model for the data under study;
2) to use that model to supply answers to whatever it is that we want to know.

This process is necessarily ongoing, as seldom is a model ever ideal; rather, model building, like the pursuit of happiness, is something that one should always be working on, continually examining residuals and worrying about other variables.

In the present study, we construct seasonal ARIMA models for time series and show how they can be used to estimate the trend (or growth rate) of the series. While the various aspects of a series' behavior quite likely result from forces not entirely captured by ARIMA models, we believe that this is a natural place to begin. Having first constructed an ARIMA model, we can then allow for outliers or interventions, or relate two or more time series. The need for this more elaborate analysis is suggested both by an examination of the white noise residuals — the sequence of one-step-ahead forecast errors — and by economic or institutional knowledge of any additional variables to which the residuals may be related. The criterion for including the results of such an analysis is whether the variance of these residuals is materially reduced.

An ARIMA model is a means for expressing the current value z_t of a time series in terms of the past values z_{t-1}, z_{t-2},... of that series. Thus we could write

$$z_t = \pi_1 z_{t-1} + \pi_2 z_{t-2} + \ldots + a_t = \Sigma \pi_j z_{t-j} + a_t \qquad (2.1)$$

where a_t is the random error (shock, innovation) at time t unpredictable from the series' past, assumed to be (iid) with mean 0 and variance σ^2. If B denotes the backshift operator defined by $B^j z_t = z_{t-j}$ then (2.1) becomes

$$(1 - \Sigma \pi_j B^j) z_t = \pi(B) z_t = a_t \qquad (2.2)$$

As this expression implies a potentially infinite number of parameters we assume, for parsimony and simplicity, that $\pi(B)$ can be expressed as a ratio $\Phi(B)/\theta(B)$, whereupon multiplying (2.2) through by $\theta(B)$ we obtain

$$\Phi(B) z_t = \phi(B) a_t. \qquad (2.3)$$

In practice z_t is frequently nonstationary but stationary after differencing the series d times, in which case

$$\Phi(B) = \Delta(B)\theta(B) \qquad (2.4)$$

where $\Delta(B) = (1-B)^d$. If $\phi(B)$ and $\theta(B)$ are of degrees p and q then (2.3) is referred to as an autoregressive-integrated moving average (ARIMA) model of order (p,d,q). In the model (2.3), the roots of the characteristic equations $\phi(B) = 0$ and $\theta(B) = 0$ lie outside the unit circle, and the zeroes of the "differencing" polynomial $\Delta(B)$ lie on the unit circle. The differenced series $w_t = \Delta(B) z_t$ is therefore a stationary, invertible autoregressive-moving average processes

$$w_t = \sum_{j=1}^{p} \phi_j w_{t-j} - \sum_{j=1}^{q} \theta_j a_{t-j} + a_t. \qquad (2.5)$$

In constructing a model for an observed time series it is essential to have available means for identification (of the particular model form), for

estimation of this form, and for diagnostic checking, or model criticism, after fitting. For time series this procedure is discussed in Box and Jenkins (1970, Ch. 6-8).

3. SEASONAL MODEL AND TREND ESTIMATION

As indicated in Section 2, a model for a series or system which captures the essential features of that process may be used to provide answers to the major questions of this investigation. Here we are interested in the use of a model fitted to a time series to construct estimates of that series' current trend or rate of growth. These quantities will be seen to depend on forecasts of the series' future values, and thus this section begins by analyzing forecast functions for ARIMA models, particular for seasonal ARIMA models, extending Chapters 5 and 9 of (Box and Jenkins, 1970). We then present, in Section 3.3, the general trend estimation procedure.

3.1 Forecast Function for ARIMA Models

Given a segment z_t, $z_{t-1}, \ldots$ of a time series, the forecast of a future value $z_{t+\ell}$ of that series with minimum mean square error is the conditional expectation

$$\hat{z}_t(\ell) = E_t(z_{t+\ell}) = E(z_{t+\ell} | z_t, z_{t-1}, \ldots). \tag{3.1}$$

For the model (2.3) this forecast function (considered as a function of ℓ, the "lead time") is known to satisfy the difference equation

$$\Phi(B)\hat{z}_t(\ell) = \theta(B)\hat{a}_t(\ell) \tag{3.2}$$

where B now operates on ℓ, and

$$\hat{a}_t(\ell) = \begin{cases} a_{t+\ell}, & \ell \leq 0 \\ 0, & \ell > 0. \end{cases}$$

In particular,

$$\Phi(B)\hat{z}_t(\ell) = 0, \quad \ell > q. \tag{3.3}$$

For $\ell > q - P$, eq. (3.2) has a general solution of the form

$$\hat{z}_t(\ell) = \sum_{i=1}^{P} b_i^{(t)} f_i(\ell), \qquad (3.4)$$

where the $f_i(\ell)$, $1 \leq i \leq P$, are functions (in general polynomials, exponentials, sines and cosines and products of these) of the lead time ℓ; in particular, $f_i(\ell) = g_i^{\ell}$ if the root g_i^{-1} of $\Phi(B) = 0$ is real and distinct.

Having determined the functions $f_i(\ell)$, the solution (3.4) further requires a knowledge of the adaptive coefficients $b_1^{(t)}, \ldots, b_P^{(t)}$, so named because they change with the forecast origin t, adapting to new information (data) continually becoming available. In Appendix 1 it is shown, in turn, how to obtain the $\{b_i^{(t)}\}$:

(a) from an initial set of forecasts,

(b) directly from the available observations $\{z_t, z_{t-1}, \ldots\}$,

(c) from their previous values as additional observations become available and the forecast origin is shifted forward.

3.2 Seasonal ARIMA Models

An important case of the ARIMA model (2.3) found useful in analyzing seasonal time series, which exhibit periodic behavior of a stochastic or adaptive nature, is the multiplicative seasonal model. Suppose that in (2.3) the operators $\Phi(B)$ and $\theta(B)$ factor according to

$$\Phi(B) = \Phi_1(B)\Phi_2(B^s)$$

$$\theta(B) = \theta_1(B)\theta_2(B^s)$$

where the seasonal operators Φ_2 and θ_2 are functions of B^s, s denoting the period (e.g., s=12 for monthly data with an annual seasonal pattern). Then the model (2.3) may be written

$$\Phi_1(B)\Phi_2(B^s)z_t = \theta_1(B)\theta_2(B^s)a_t. \qquad (3.5)$$

A simple example of a seasonal ARIMA model would be

$$z_t = z_{t-12} + a_t - \theta a_{t-1},$$

or

$$(1 - B^{12})z_t = (1 - \theta B)a_t,$$

in which $\Phi_2(B) = 1 - B^{12}$, $\theta(B) = 1 - \theta B$, and $\Phi_1(B) = \theta_2(B) = 1$. This model expresses the current observation as the sum of last year's value and a linear combination of this and last month's shocks.

Typically Φ_1 and θ_1 will be of low order. As in (2.3), $\Phi_1(B)$ incorporates nonseasonal differencing operators applied to render the series stationary. For example, the model fitted to the Demand Deposit series in Section 4 is of the form

$$(1 - B)(1 - B^{12})z_t = (1 - \theta B)(1 - \theta B^{12}) \qquad (3.6)$$

and has [noting $1 - B^{12} = (1 - B)(1 + B + \dots + B^{11})$]

$$\Phi_1(B) = (1 - B)^2, \ \Phi_2(B) = \sum_{j=0}^{11} B^j. \qquad (3.7)$$

3.3 Estimated Trend

The general procedure for trend estimation for a seasonal time series can now be set forth. In the seasonal ARIMA model (3.5), if Φ_1, Φ_2, θ_1 and θ_2 are respectively of degrees p_1, p_2, q_1 and q_2 in B, then analogous to equation (3.4) the eventual forecast function for the model (3.5) may be written in the form

$$\hat{z}_t(\ell) = \sum_{i=1}^{p_1} b_{1i}^{(t)} f_{1i}(\ell) + \sum_{i=1}^{p_2} b_{2i}^{(t)} f_{2i}(\ell)$$

$$= T_t(\ell) + S_t(\ell) \qquad (3.8)$$

The second sum on the right hand side of (3.8) is associated with the seasonal part of the model, determined by Φ_2 and θ_2. The first sum $T_t(\ell)$ embodies all systematic nonseasonal influences, and may thus be regarded as the trend. Our proposed trend estimate for seasonal time series described by the model (3.5) is therefore

$$T_t(\ell) = \hat{z}_t(\ell) - S_t(\ell) = \sum_{i=1}^{p_1} b_{1i}^{(t)} f_{1i}(\ell) \qquad (3.9)$$

The quantity $T_t(\ell)$, which incorporates anything systematic and nonseasonal that is known about the series at the current time t, will often correspond well to our instinctive notion of trend. For example if $\Phi(B)$ contains a factor $(1 - B)^d$ reflecting a nonstationarity removed by differencing the series d times, then $T_t(\ell)$ contains a polynomial of degree d-1 (in ℓ), with coefficients $b_{1i}^{(t)}$ adapting with the origin t. Thus if d is greater than 1 (equal to or greater than 1 if a constant term also appears in the expression for $(1 - B)z_t$) then at least a linear trend will be incorporated into $T_t(\ell)$. On the other hand, we would not ordinarily think of a series z_t which is already stationary as possessing a trend, and indeed $T_t(\ell)$ for such a series would either be zero or possess only damped terms based on the roots of the stationary autoregressive operator $\phi(B)$.

Similar statements can be made about $S_t(\ell)$; for example, whenever $\Sigma_0^{s-1} B^j$ is a factor of $\Phi_2(B)$, $S_t(\ell)$ will include a periodic function of period s; this is the case for the model (3.6).

4. ANALYSIS OF DEMAND DEPOSIT SERIES

The U.S. Money Supply is a closely watched and much investigated series in economic policy making and analysis. Changes in the underlying behavior of this series have an immediate impact on financial markets and are felt to have a strong association with many measures of economic activity, including income, inflation and employment. It is therefore of prime importance for both the Federal Reserve Board and the public to have available the best and most timely information possible on the current behavior of this series, i.e., its current trend or rate of growth. A key element of the money supply is its demand-deposit component, and this section is devoted to estimating the current rate of growth of this series.

4.1 Airline Model

The model fitted to the Demand Deposit series (Section 4.2) is of the general form (3.6), which has been found to describe quite well a number of seasonal time series. It was first fitted by Box and Jenkins (Chapter 9) to a series of logged monthly passenger totals in international air travel and has thus become known as the Airline model. This model has a forecast functio $\hat{z}_t(\ell)$ satisfying the difference equation

$$(1 - B)(1 - B^{12})\hat{z}_t(\ell) = (1 - \theta B)(1 - \theta B^{12})\hat{a}_t(\ell) \qquad (4.1)$$

where $\hat{a}_t(\ell)$ is as below (3.2). This equation has a solution which can be expressed in several equivalent forms, involving dummy variables or sines and cosines for the seasonal term $S_t(\ell)$ in (3.8). These are derived in part by Box and Jenkins (1970) and in part in Appendix 2, where it is found that the most appropriate expression for our current use is of the form

$$\hat{z}_t(\ell) = b_o^{(t)} + b_m^{(t)} + b^{(t)}\ell \ . \qquad (4.2)$$

The three terms on the right hand side of (4.2) are respectively the level, seasonal and slope components of the forecast. Therefore the estimated trend is, as in (3.9),

$$T_t(\ell) = \hat{z}_t(\ell) - b_m^{(t)} = b_o^{(t)} + b^{(t)}\ell \tag{4.3}$$

where in (4.2) and (4.3) m = 1,2,..,12 indexes the month of the year in which the value $z_{t+\ell}$ falls. There are thus 14 coefficients $(b_1^{(t)},\ldots,b_{12}^{(t)},b_o^{(t)}, b^{(t)})$ the first 12 of which satisfy the constraint $\Sigma\, b_i^{(t)} = 0$. As with the coefficients in nonseasonal ARIMA models these may also be determined in several ways, as illustrated in Appendix 2. In particular the following results are of primary interest:

1. The slope coefficient $b^{(t)}$ in (4.2) is given by

$$b^{(t)} = [\hat{z}_t(13) - \hat{z}_t(1)]/12, \tag{4.4}$$

which is, as perhaps expected, the average monthly increment to the forecasts. The other b-coefficients are similarly determined from the forecasts $\hat{z}_t(1),\ldots, \hat{z}_t(13)$.

2. The b's are linear combinations of forecasts which are themselves linear combinations of the data z_t, $z_{t-1},\ldots$. Thus the coefficients b, b_o, $b_1,\ldots$, b_{12} are expressable directly in terms of this available data, as

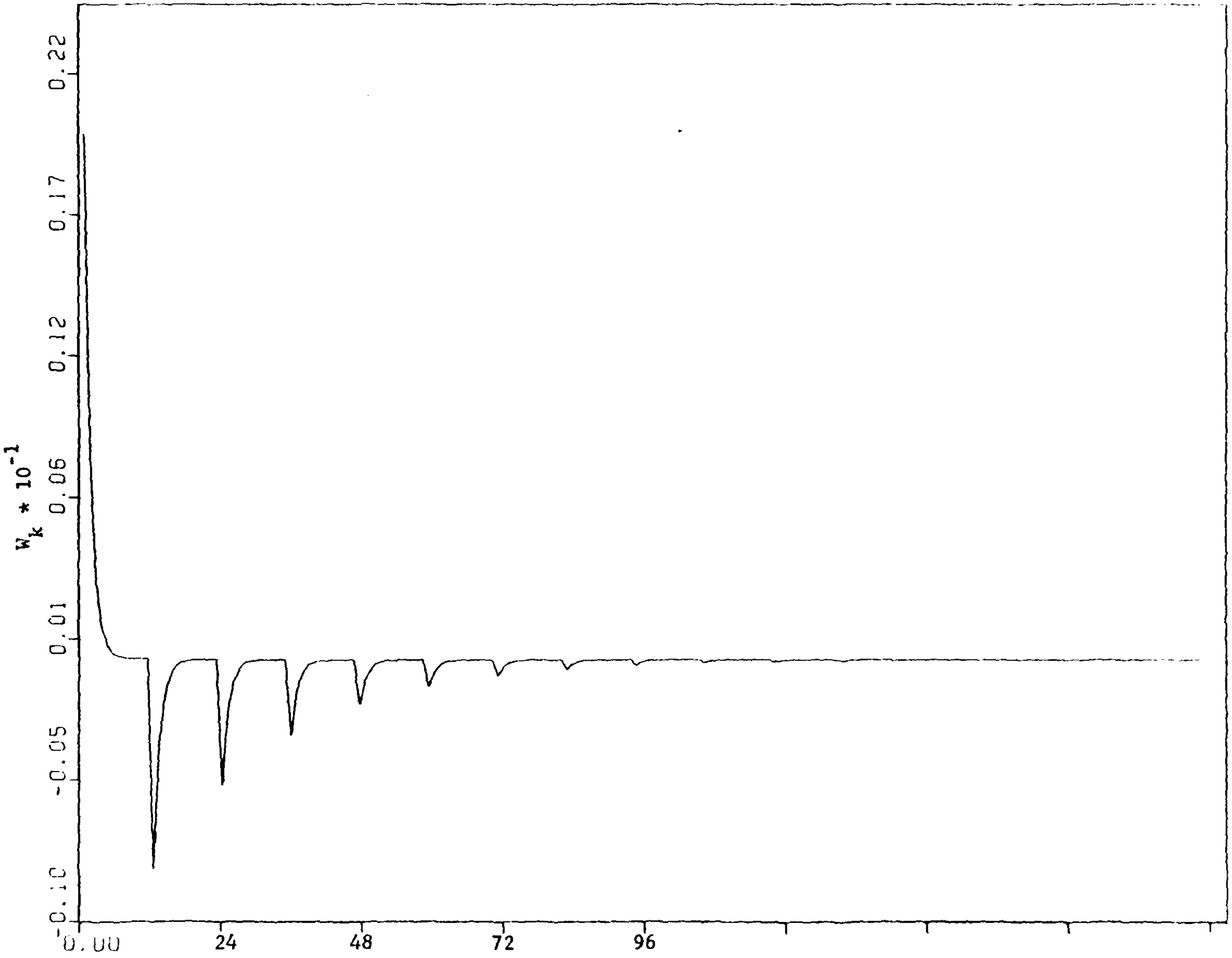
$$b_i^{(t)} = W_i(B)z_t. \tag{4.5}$$

For example, Figure 1 shows the weights $\{W_j\}$ in determining the slope

$$b^{(t)} = W(B)z_t = \Sigma W_j z_{t-j}, \tag{4.6}$$

for the Airline data referred to above.

Figure 1. Weights $\{W_k\}$ in $b_t = \Sigma W_k z_{t-k}$ for slope coefficient $b^{(t)}$ in Airline model.



3. The coefficients of the forecast function evolve as new observations result in a shifting forward of the forecast origin, t; for example, the relation

$$b^{(t+1)} = b^{(t)} + (1 - \theta)(1 - \Theta)a_{t+1} \tag{4.7}$$

shows how a fraction of the new information (innovation, shock) a_{t+1} is incorporated into the revised slope estimate.

4.2 ARIMA Model for Demand Deposit Series

Monthly observations on the demand-deposit component of the money supply (DD) were obtained for the period 1/69 - 1/78 (Source: Federal Reserve Bulletin. The data are periodically revised, the series here used being as of 10/78). Application of the usual model fitting procedure (Box and Jenkins. 1970, Ch. 6-8) to the logged series ($z_t = \log DD_t$) yielded a fitted ARIMA model of the form

$$\Delta\Delta_{12}z_t = (1 - .6B^{12})(1 + .1B)a_t \tag{4.8}$$

with $\sigma_a = .0058$. This is the Airline model form, with parameter estimates $\hat{\Theta} = .6$, $\hat{\theta} = -.1$, so that the trend in the logged demand deposit series can be estimated as in Section 3.3. We shall focus on the first difference of this trend, which is the estimated rate of growth (deseasonalized or seasonally adjusted) of demand deposits. It is in fact the rates of growth of money supply series which are most often examined both within the Federal Reserve and in financial communities.

4.3 Estimated Rate of Growth

Taking the first difference of the trend function (4.3) shows that the estimated current (at time origin t) rate of growth of demand deposits is simply the slope coefficient $b^{(t)}$. Thus, by virtue of eq. (4.4) the practical

application of the concepts herin proposed reduces in this case simply to computing the difference between the lead-13 and lead-1 forecasts of log DD. As it is customary to express these figures in terms of annual rates, we omit the division by 12.

Figure 2 shows the weights W_k which can be used to calculate $b^{(t)}$ from the series values z_t, $z_{t-1}, \ldots$, as in equation (4.6) (without the division by 12 in (A2.11)). This is broadly similar to Figure 1 for the airline data, differences resulting from the first order MA parameter.

The calculation of the estimated growth rate $b^{(t)}$ for DD was made over the period 8/72 - 9/78, and the results are shown in Table 1 and Figure 3.

4.4 Error Limits of Growth-Rate Estimates

Since the slope estimate $b^{(t)}$ and the other coefficients $b_i^{(t)}$ are functions of the forecasts, the errors in these adaptive coefficients are the corresponding functions of the forecast errors $e_t(\ell) = z_{t+\ell} - \hat{z}_t(\ell)$. In particular the error corresponding to $b^{(t)}$ in (4.4) is

$$\begin{aligned} e^{(t)} &= [e_t(13) - e_t(1)] \\ &= [a_{t+13} + \psi_1 a_{t+12} + \cdots + \psi_{11} a_{t+2} \\ &\qquad + (\psi_{12} - 1) a_{t+1}] \end{aligned} \tag{4.9}$$

with mean square

$$\sigma^2[e^{(t)}] = [\Sigma_0^{11} \psi_j^2 + (\psi_{12} - 1)^2]. \tag{4.10}$$

Letting the actual growth rates be defined by

$$\beta^{(t)} = z_{t+13} - z_{t+1}$$

also gives

$$e^{(t)} = b^{(t)} - \beta^{(t)}.$$

Figure 2. Weights $\{W_k\}$ in $b_t = \sum_{k\geq 0} W_k z_{t-k}$ for Demand Deposit Model.

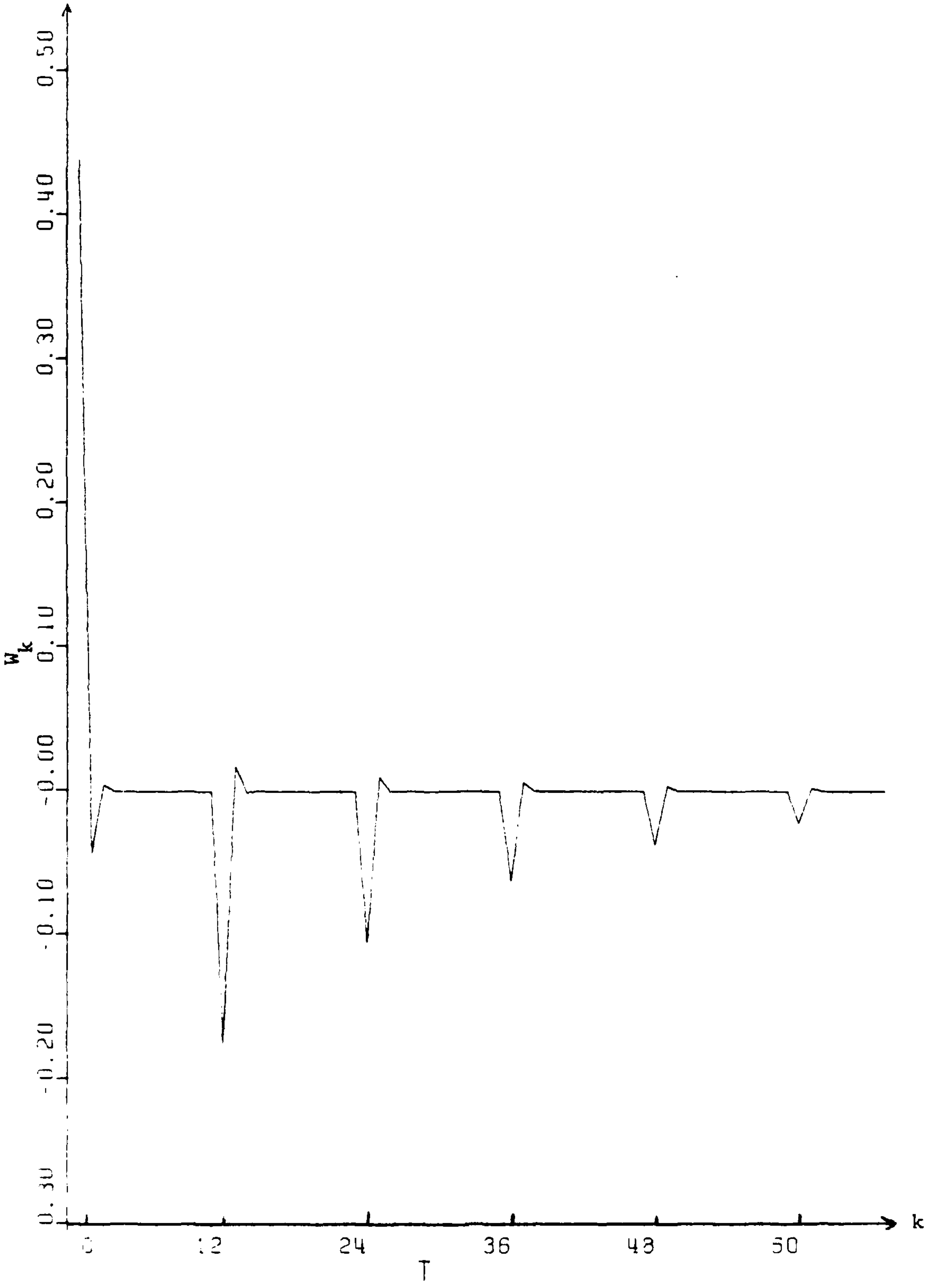


Table 1. Log Demand Deposits, 1972-1978

t	$\hat{z}_t(13)$	$\hat{z}_t(1)$	$b^{(t)}$	$\beta^{(t)}$	z_{t+13}	z_{t+1}	$e^{(t)}$	$\hat{a}_t$
7208	12.22	12.166	.058244	.055041	12.202	[illegible]	[illegible]	[illegible]
7209	12.232	12.178	.058432	.052003	12.232	[illegible]	[illegible]	[illegible]
7210	12.247	12.139	.058194	.058035	12.28	12.199	[illegible]	[illegible]
7211	12.277	12.213	.059211	.05171	12.281	12.23	-.0075001	[illegible]
7212	12.304	12.24	.063979	.042515	12.28	12.237	-.021464	[illegible]
7301	12.262	12.199	.063061	.046555	12.243	12.196	-.016502	.0074286
7302	12.266	12.204	.061985	.054472	12.249	12.195	-.007513	.0020955
7303	12.279	12.22	.058106	.054141	12.271	12.217	-.0039651	-.0021676
7304	12.249	12.192	.056882	.042803	12.243	12.2	-.014073	-.006635
7305	12.277	12.217	.060017	.041028	12.265	12.224	-.018989	.0027167
7306	12.296	12.233	.063011	.040767	12.272	12.232	-.022245	-.001233
7307	12.287	12.225	.06236	.039553	12.258	12.219	-.022802	-.0050174
7308	12.293	12.233	.059821	.042377	12.265	12.223	-.017444	-.0031001
7309	12.288	12.232	.055875	.040506	12.272	12.232	-.015369	-.002107
7310	12.298	12.242	.055718	.034684	12.284	12.25	-.021034	-.001258
7311	12.339	12.281	.05876	.029736	12.311	12.281	-.029024	-.0044805
7312	12.35	12.291	.059072	.020919	12.301	12.28	-.038153	-.015161
7401	12.295	12.24	.054843	.015313	12.258	12.243	-.03953	-.0041493
7402	12.303	12.247	.055614	.017847	12.267	12.249	-.037968	.0037913
7403	12.33	12.274	.056652	.017847	12.289	12.271	-.038805	-.0012999
7404	12.305	12.249	.055785	.02575	12.268	12.243	-.030036	.0039213
7405	12.316	12.263	.053131	.033391	12.299	12.265	-.019741	.0092712
7406	12.328	12.274	.054218	.03228	12.305	12.272	-.021938	-.001853
7407	12.317	12.263	.053723	.035999	12.294	12.258	-.017724	7.083E-4
7408	12.32	12.268	.051716	.038584	12.304	12.265	-.013132	7.2426E-4
7409	12.325	12.274	.050476	.031565	12.304	12.272	-.01891	-.0092823
7410	12.335	12.286	.049633	.035723	12.32	12.284	-.01391	.0034027
7411	12.365	12.316	.04913	.029319	12.34	12.311	-.019811	-.009092
7412	12.363	12.316	.047338	.035653	12.337	12.301	-.011684	-.0027626
7501	12.303	12.262	.041273	.041723	12.3	12.258	4.5521E-4	.0035857
7502	12.303	12.263	.039614	.038525	12.306	12.267	-.0010888	-9.2895E-4
7503	12.332	12.291	.04113	.047526	12.337	12.289	.0063957	.008221
7504	12.305	12.264	.04061	.04307	12.311	12.268	.0024597	-.0021032
7505	12.332	12.29	.042179	.027916	12.327	12.299	-.014262	-.009591
7506	12.353	12.307	.045887	.030173	12.335	12.305	-.015714	.0011447
7507	12.339	12.293	.045146	.031945	12.326	12.294	-.013201	.0021974
7508	12.349	12.303	.045429	.028854	12.333	12.304	-.016575	-.0026567
7509	12.358	12.312	.045719	.046129	12.35	12.304	4.1004E-4	.012306
7510	12.359	12.317	.042406	.037825	12.358	12.32	-.0045806	-.0062619
7511	12.393	12.35	.043767	.049999	12.39	12.34	.0062317	.0067182
7512	12.379	12.339	.04013	.05459	12.391	12.337	.01446	.0029339
7601	12.335	12.296	.039025	.05028	12.35	12.3	.011255	-.0021589
7602	12.347	12.306	.04046	.054593	12.36	12.306	.014134	.0037541
7603	12.369	12.329	.040088	.061657	12.399	12.337	.021569	.011997
7604	12.357	12.314	.043376	.056554	12.368	12.311	.013178	-.0063648
7605	12.379	12.336	.042535	.062817	12.39	12.327	.020282	5.0813E-4
7606	12.372	12.334	.038699	.072413	12.407	12.335	.03372	.010283
7607	12.363	12.324	.039157	.071776	12.398	12.326	.032619	6.7597E-4
7608	12.375	12.335	.040036	.075984	12.409	12.333	.035949	.0026142
7609	12.377	12.333	.038973	.072183	12.422	12.35	.033211	.0035827
7610	12.408	12.364	.043895	.07112	12.429	12.358	.027224	-.004821
7611	12.425	12.384	.04139	.070647	12.461	12.39	.029256	.003558
7612	12.432	12.388	.044079	.072724	12.464	12.391	.029646	.0039372
7701	12.397	12.352	.045251	.067029	12.417	12.35	.021778	-.0069894
7702	12.401	12.356	.044388	.062011	12.422	12.36	.017623	-.0027653
7703	12.432	12.387	.04569	.069426	12.468	12.399	.023526	.014613
7704	12.425	12.374	.050689	.074409	12.442	12.348	.02372	.0011639
7705	12.437	12.389	.048143	.074175	12.464	12.39	.026033	[illegible]
7706	12.445	12.397	.048346	.065577	12.476	12.407	.02023	[illegible]
7707	12.45	12.397	.052461	.070162	12.468	12.398	.017701	[illegible]
7708	12.459	12.406	.052732	.074342	12.483	12.409	.02141	.005748
7709	12.472	12.419	.053777	.067405	12.49	12.422	.012627	[illegible]
7710	12.489	12.434	.05521	0	0	12.429	0	
7711	12.511	12.456	.053282	0	0	12.461	0	
7712	12.515	12.46	.054705	0	0	12.464	0	
7801	12.48	12.424	.05624	0	0	12.417	0	
7802	12.473	12.425	.053444	0	0	12.422	0	
7803	12.506	12.453	.052338	0	0	12.445	0	
7804	12.490	12.441	.058184	0	0	12.442	0	
7805	12.532	12.464	[illegible]	0	0	12.464	0	
7806	12.534	12.475	.058675	0	0	12.476	0	
7807	12.525	12.466	[illegible]	0	0	12.463	0	
7808	12.537	12.477	.059704	0	0	12.453	0	
7809	12.556	[illegible]	.062203	0	0	[illegible]	0	

Figure 3. Estimated and Actual Demand Deposit Growth Rates with Standard Error Limits for Estimated Rates

□ $s^{(t)}$ × $b^{(t)}$ .... $b^{(t)} \pm$ 1 S.E.

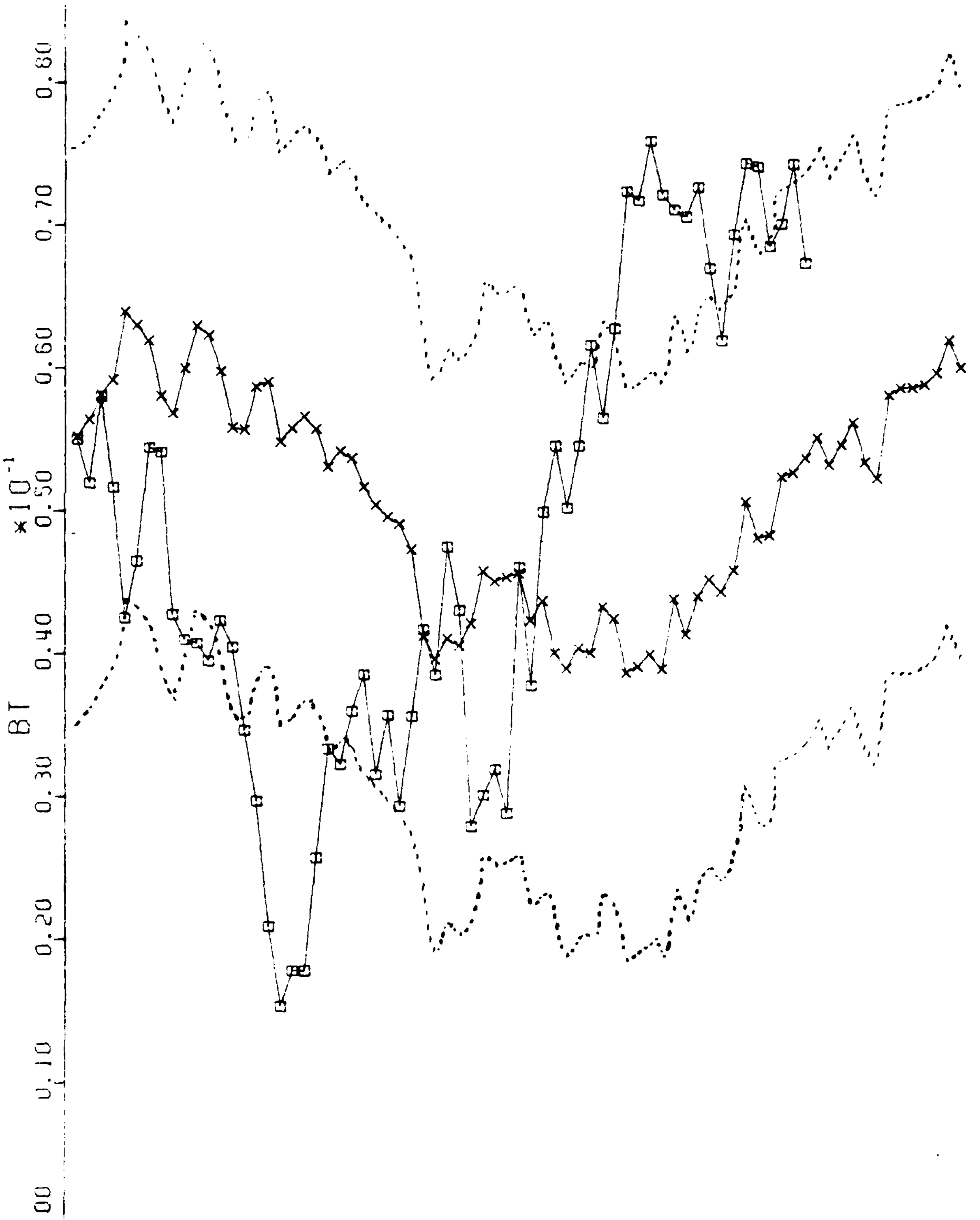


Table 1 and Figure 3 display these actual growth rates $\beta^{(t)}$ and errors $e^{(t)}$. The dotted lines in Figure 3 are standard error limits $\pm e^{(t)}$ associated with the estimated rates of change. Based on (4.10) we have

$$E[b^{(t)} - \beta^{(t)}]^2 = .00040, \tag{4.11}$$

making the standard error .020 or 2%. (Recall that these are annualized figures; the monthly standard error itself is (1/12)th of this amount.) This mean square error was obtained from the fitted model (4.8), and indeed one of the virtues of the model building approach is that such internal error limits are thereby available. But it is also of interest that the empirical MSE over this period,

$$\frac{1}{61}\Sigma[b^{(t)} - \beta^{(t)}]^2 = .00044, \tag{4.12}$$

is in close agreement with the model-based estimate.

Figure 3 also illustrates the limitations of the present model and the importance of making the model building effort an ongoing one, to incorporate additional relevant information. To give an estimate of (say) 6% as the current seasonally adjusted rate of increase of demand deposits but to have the 95% confidence interval span the range 2 to 10% is uncomfortable, even though other money supply estimates/forecasts typically have errors of comparable magnitudes [e.g., Porter et al. (1978), Pierce et. al. (1981)]. Ths use of other variables related to causes of the shortfall of money growth during 1974 (relative to its estimate in Figure 3) or of the overshooting of money growth in 1976-77 could possibly reduce this error.

5. CONCLUSIONS

We have presented a method for estimating the current trend or rate of growth of seasonal time series, based on models for those series, and have illustrated its application with the demand deposit component of the U.S. Money Supply. The resulting trend estimates are thus derived in an optimal manner from the underlying characteristics of the series and consequently do not suffer from the arbitrariness inherent in methods not so derived. But we have also noted the need to regard such model building investigations as ongoing pursuits; in particular, the incorporation of outliers, interventions and other variables into the model should reduce the mean square error of the resulting trend or growth-rate estimates.

APPENDIX 1. ADAPTIVE COEFFICIENTS IN ARIMA FORECASTS AND TREND ESTIMATES

A.1 Determination of Adaptive Coefficients from Initial Forecasts

For lead times $\ell > q$, the forecast function $\hat{z}_t(\ell)$ satisfies the homogenous difference equation (3.3), and thus for such ℓ (3.4) holds for arbitrary constants $b_i^{(t)}$. These constants are determined (uniquely) by a set of initial conditions which are equivalent to assuming that (3.4) [though not (3.3)] also holds for $\ell = q-P+1, \ldots, q-1, q$, which, given $\hat{z}_t(\ell)$ for these ℓ values [$\hat{z}_t(\ell) = z_{t+\ell}$ if $\ell \leq 0$], defines a set of P equations in the P unknowns $b_1^{(t)}, \ldots, b_P^{(t)}$. In fact, any set of P consecutive forecasts/actuals

$$\underset{\sim}{\hat{z}}(\ell) = [\hat{z}_t(\ell), \hat{z}_t(\ell)+1), \ldots, \hat{z}_t(\ell+P-1)]' \qquad \text{(A1.1)}$$

such that $\ell > q-P$ can be used. Letting

$$\underset{\sim}{b}(t) = [b_1^{(t)}, \ldots, b_P^{(t)}]' \quad \text{and}$$

$$\underset{\sim}{F}_\ell = \begin{bmatrix} f_1(\ell) & \ldots & f_P(\ell) \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ f_1(\ell+P-1) & \ldots\ldots & f_P(\ell+P-1) \end{bmatrix} \qquad \text{(A1.2)}$$

this system can be written in the form

$$\underset{\sim}{\hat{z}}_t(\ell) = \underset{\sim}{F}_\ell \underset{\sim}{b}^{(t)}, \qquad \text{(A1.3)}$$

whence

$$\underset{\sim}{b}^{(t)} = \underset{\sim}{F}_\ell^{-1} \, \underset{\sim}{\hat{z}}_t(\ell). \qquad \text{(A1.4)}$$

If $q \leq P$, it is often convenient to consider the system in terms of the first P forecasts ($\ell = 1$), so that, with $\hat{\underset{\sim}{z}}_t^{(1)} = \hat{\underset{\sim}{z}}_t$,

$$\underset{\sim}{b}^{(t)} = \underset{\sim}{F}_1^{-1}\, \hat{\underset{\sim}{z}}_t. \qquad \text{(A1.5)}$$

As an example, consider the AR(1) process

$$(1 - \phi B)x_t = a_t$$

for which (3.4) is

$$\hat{z}_t(\ell) = f(\ell)\; b^{(t)} = \phi^\ell z_t, \quad \ell \geq 0,$$

so that

$$\underset{\sim}{F}_\ell = f(\ell) = \phi^\ell ,$$

and the calculation (A1.4),

$$b^{(t)} = (\phi^\ell)^{-1}\; \hat{z}_t(\ell) = z_t,$$

can be made for any $\ell > q-P = -1$.

A1.2 Adaptive Coefficients in Terms of the Observed Series

Since the coefficients $\underset{\sim}{b}^{(t)}$ are linear combinations of an initial set of forecasts [Eq. (A1.4)], and since the forecasts at origin t are linear combinations of the series $\{z_t, z_{t-1}, \ldots\}$ (Whittle, 1963, Box and Jenkins, 1970), it follows that the coefficients are expressable as linear combinations of the series. In particular, writing

$$\hat{z}_t(\ell) = \pi^{(\ell)}(B) z_t, \qquad \text{(A1.6)}$$

it follows that

$$\pi^{(\ell)}(B) = \pi(B)\Big[\sum_{j=\ell}^{\infty} \psi_j B^{j-\ell}\Big], \qquad \text{(A1.7)}$$

where

$$\pi(B) = \Sigma\ \pi_j B^j = \psi^{-1}(B) = (\sum_{o}^{\infty} \psi_j B^j)^{-1}$$

Hence, if

$$\underset{\sim}{\pi}^{(\ell)}(B) = [\pi^{(\ell)}(B), \ldots, \pi^{(\ell+P-1)}(B)]' \qquad (A1.8)$$

then from (A1.4) and (A1.6), the desired expression for the coefficients is

$$\underset{\sim}{b}^{(t)} = \underset{\sim}{W}(B) z_t \qquad (A1.9)$$

where

$$\underset{\sim}{W}(B) = \underset{\sim}{F}_{\ell}^{-1}\ \underset{\sim}{\pi}^{(\ell)}(B) = [W_1(B), \ldots, W_P(B)]' \ . \qquad (A1.10)$$

Of course the weights W(B) are the same for any value of $\ell > q-P$ in (A1.10).

For the AR(1) process introduced at the end of Section 3.1,

$$\pi^{(\ell)}(B) \equiv \phi^{\ell},\ W(B) \equiv 1.$$

A1.3 Updating

Given a new observation z_{t+1}, the forecasts from origin t+1 may be expressed as simple adjustments to the forecasts calculated at origin t, the adjustments depending on (and only on) the new information a_{t+1}. Thus the adaptive coefficients may be so updated. In particular, the usual updating formula

$$\hat{z}_{t+1}(\ell) = \hat{z}_t(\ell+1) + \psi_{\ell}\ a_{t+1}$$

is, in the previous notation,

$$\underset{\sim}{\hat{z}}_{t+1}^{(\ell)} = \underset{\sim}{\hat{z}}_{t}^{(\ell+1)} + \underset{\sim}{\psi}_{\ell}\ a_{t+1}, \qquad (A1.11)$$

with $\psi_{\ell} = (\psi_{\ell}, \psi_{\ell+1}, \ldots, \psi_{\ell+P-1})'$, so that, from (A1.3) and its counterpart at time t+1,

$$\underset{\sim}{F}_{\ell}\, b^{(t+1)} = \underset{\sim}{F}_{\ell+1}\underset{\sim}{b}^{(t)} + \underset{\sim}{\psi}_{\ell} a_{t+1} \qquad (A1.12)$$

whence

$$\begin{aligned}\underset{\sim}{b}^{(t+1)} &= \underset{\sim}{F}_{\ell}^{-1}\underset{\sim}{F}_{\ell+1}\underset{\sim}{b}^{(t)} + F_{\ell}^{-1}\underset{\sim}{\psi}_{\ell} a_{t+1} \\ &= \underset{\sim}{L}\,\underset{\sim}{b}^{(t)} + \underset{\sim}{k}\, a_{t+1}. \qquad (A1.13)\end{aligned}$$

Again considering the AR(1) process at the end of Section 3.1, (A1.13) becomes, taking $\ell = 0$ so that $F_0 = 1$ and $F_1 = \phi$,

$$b^{(t+1)} = \phi b^{(t)} + a_{t+1}$$

which, since $b^{(t)} = z_t$, is simply the equation for the process.

A1.4 "Error" of Adaptive Coefficient Estimates

Since the various $b^{(t)}$'s are functions of the forecasts, the errors in these adaptive coefficients may be defined as the corresponding functions of the forecast errors. In general,

$$\underset{\sim}{e}[\underset{\sim}{b}^{(t)}] = \underset{\sim}{F}^{-1}\, \underset{\sim}{e}_{t}^{(\ell)} = \underset{\sim}{\beta}^{(t)} - \underset{\sim}{b}^{(t)} \qquad (A1.14)$$

where, analogous to (A1.4),

$$\underset{\sim}{\beta}^{(t)} = \underset{\sim}{F}_{\ell}^{-1}(z_{t+1}, \ldots, z_{t+\ell+p-1})' . \qquad (A1.15)$$

Note that $\underset{\sim}{\beta}^{(t)}$ and $\underset{\sim}{e}[\underset{\sim}{b}^{(t)}]$, unlike $\underset{\sim}{b}^{(t)}$ itself, depend on the value of ℓ.

APPENDIX 2: AIRLINE MODEL TREND ESTIMATION

The solution of the forecast function (4.1) for the airline model (3.6) is of the form

$$\hat{z}_t(\ell) = b^{(t)} + b_1^{(t)}\ell + \sum_{j=1}^{6} [b_{1j}^{(t)} \cos \frac{2\pi j\ell}{12} + b_{2j}^{(t)} \sin \frac{2\pi j\ell}{12}].$$

An equivalent form, which is easier to work with, uses monthly dummy or indicator variables, and there are several variants. Adapting the development in [Box and Jenkins (1970), p. 309], it can be shown that (A2.1) can be written in the alternative form

$$\hat{z}_t(\ell) = b_{om}^{(t)} + b^{(t)}\ell \qquad (A2.2)$$

where ℓ is measured in months.

Next, note that the monthly coefficient $b_{om}^{(t)}$ incorporates both an overall level effect

$$b_o^{(t)} = \frac{1}{12} \sum_{m=1}^{12} b_{om}^{(t)} \qquad (A2.3)$$

and a seasonal effect specific to the m^th month

$$b_m^{(t)} = b_{om}^{(t)} - b_o^{(t)}; \qquad (A2.4)$$

thus, the forecast (A2.2) is equivalently equation (4.2) of the text, from which the trend (4.3) is derived.

Parallel to Appendix 1 we examine each group of coefficients, obtaining expressions for them (a) from an initial set of forecasts, (b) from the available observations z_t, $z_{t-1}, \ldots$, and (c) from their previous values as new observations become available and the origin is shifted forward.

A2.1 Initial Determination of Adaptive Coefficients

As in (A1.1) let $\hat{\underset{\sim}{z}}_t^{(\ell)}$ denote the vector of P=13 forecasts $\hat{z}_t(\ell)$, ..., $\hat{z}_t(\ell+12)$, and for convenience let us take $\ell = 1$, letting $\hat{\underset{\sim}{z}}_t^{(1)} = \hat{\underset{\sim}{z}}_t$, though any positive ℓ would suffice. Then (A1.3) is a system of 13 equations of the form (A2.2) which needs to be solved for the 13 unknowns $b_{o1}^{(t)}, \ldots, b_{o,12}^{(t)}$, $b^{(t)}$. This is most immediately done by noting that the equations for $\hat{z}_t(1)$ and $\hat{z}_t(13)$ imply that the slope coefficient is equation (4.4) of the text.

In terms of $b^{(t)}$ the other 12 equations then each have solutions

$$b_{om}^{(t)} = \hat{z}_t(m) - m\, b^{(t)} \tag{A2.5}$$

giving the m^th monthly coefficient as an adjustment to the lead-m forecast, the correction being the amount required to offset the trend component of the forecast.

Given (4.4) and (A2.5), the level and seasonal effect coefficients $b_o^{(t)}$ and $b_m^{(t)}$ in (4.2) are determined as in equations (A2.3) and (A2.4):

$$b_o^{(t)} = \frac{1}{12} \sum_{m=1}^{12} \hat{z}_t(m) - \frac{13}{24}\left[\hat{z}_t(13) - \hat{z}_t(1)\right], \tag{A2.6}$$

$$b_m^{(t)} = \hat{z}_t(m) = \frac{1}{12} \sum_{n=1}^{12} \hat{z}_t(n) + \frac{13-2m}{24}\left[\hat{z}_t(13) - \hat{z}_t(1)\right]. \tag{A2.7}$$

A2.2 Adaptive Coefficients in Terms of Available Data

As in Appendix 1, the various adaptive coefficients are each expressable as linear combinations of the observed series z_t, $z_{t-1}, \ldots$. Considering first the forecasts written as in (A2.2) containing 12 monthly levels $b_{om}^{(t)}$, let

$$b_{om}^{(t)} = W_{om}(B) z_t, \quad m=1,\ldots,12 \tag{A2.8}$$

Then from (A1.6) and (A2.5),

$$W_{om}(B) = \pi^{(m)}(B) - \frac{m}{12}\,[\pi^{(13)}(B) - \pi^{(1)}(B)]; \qquad \text{(A2.9)}$$

and writing the slope coefficient as

$$b^{(t)} = W(B)z_t \qquad \text{(A2.10)}$$

which is eq. (4.6) of the text, it follows that

$$W(B) = \frac{1}{12}\,[\pi^{(13)}(B) - \pi^{(1)}(B)]. \qquad \text{(A2.11)}$$

Let us next express the forecast function as (4.2), in which the monthly level is separated into an overall level $b_o^{(t)}$ and a seasonal effect $b_m^{(t)}$ $(\Sigma_{m=1}^{12}\ b_m^{(t)} = 0)$. From (A2.6) and (A2.7), if

$$b_m^{(t)} = W_m(B)z_t, \quad m=0,1,\dots,12 \qquad \text{(A2.12)}$$

then

$$\begin{aligned} W_o(B) &= \frac{1}{12}\ \Sigma\ W_{om}(B) \\ &= \frac{1}{12}\sum_{m=1}^{12}\pi^{(m)}(B) - \frac{13}{24}\,[\pi^{(13)}(B) - \pi^{(1)}(B)] \qquad \text{(A2.13)} \end{aligned}$$

and

$$\begin{aligned} W_m(B) &= W_{om}(B) - W_o(B), \quad m=1,\dots,12 \\ &= \pi^{(m)}(B) - \frac{1}{12}\sum_{n=1}^{12}\pi^{(n)}(B) + \frac{13-2m}{24}\,[\pi^{(13)}(B) - \pi^{(1)}(B)]. \qquad \text{(A2.14)} \end{aligned}$$

Whereas the transformation from the forecasts $\hat{\underset{\sim}{z}}_t$ to the adaptive coefficients $\underset{\sim}{b}^{(t)}$ does not depend on the airline-model parameters (as the form of eventual forecast function does not), the forecasts themselves, and thus the weights such as W(B) and $W_m(B)$, are functions of θ and Θ.

These weights functions can be determined for the values $\theta = .4$ and $\Theta = .6$, corresponding to the least squares fit obtained by BJ for the airline data. Using numerical recursive calculations, we first determine

$$\begin{aligned}\pi(B) &= \psi^{-1}(B) \\ &= (1-B)\ (1-B^{12})\ (1-.4B)^{-1}\ (1-.6B^{12})^{-1} \qquad \text{(A2.15)}\end{aligned}$$

truncated at order 256, and then $\pi^{(\ell)}(B)$ for $\ell = 1,\dots,13$ is obtained as in (A1.7). Next we form the various weight functions, including W(B) which was plotted in Figure 1. Note the periodicity exhibited by this weight function, which was true of the others as well.

A2.3 Updating

As discussed in A1.3, the coefficients of the forecast function (3.4) evolve as new observations result in a shifting forward of the forecast origin. The basic formula for the calculation is equation (A1.13), with P=13. However, it is easier to work directly with the airline model form, adapting the development in [Box and Jenkins (1970), pp. 310-11] to the modified form (A2.2) of the forecast function.

Letting

$$\lambda = 1 - \theta, \ \Lambda = 1 - \Theta,$$

and letting $\ell = 12r + m$, $1 \leq m \leq 12$, then

$$\psi_\ell = \psi_{12r+m} = \lambda(1 + r\Lambda) + \delta_{12m} \qquad \text{(A2.16)}$$

where $\delta_{12m} = 1$ iff $m = 12$ and 0 otherwise. From (A2.16) and the updating formula (A1.13), it can be shown that

$$b^{(t+1)} = b^{(t)} + (\lambda\Lambda/12)a_{t+1}, \qquad \text{(A2.17)}$$

$$b_{om}^{(t+1)} = b_{o,m+1}^{(t)} + b^{(t)} + [\lambda(1 - \frac{m}{12}\Lambda + \delta_{12m}\Lambda]a_{t+1}, \qquad \text{(A2.18)}$$

where, if $m = 12$, $b_{o,13}^{(t)}$ is replaced by $b_{o1}^{(t)}$.

Thus the previously forecasted series level for each month is adjusted by the previously forecasted increment to the trend, now incorporated into the level at the new origin; and all the coefficients are adjusted by varying fractions of the new information (innovation, shock) a_{t+1}.

For the airline model coefficients $\lambda = .6$ and $\Lambda = .4$, (A2.17) and (A2.18) become

$$b^{(t+1)} = b^{(t)} + .02\ a_{t+1}$$

$$b_{om}^{(t+1)} = b_{o,m+1}^{(t)} + b^{(t)} + \begin{cases} (.6 - .02m)\ a_{t+1}, & m \neq 12 \\ .76\ a_{t+1}, & m = 12 \end{cases}$$

The seasonal effects $b_m^{(t)}$ and the level effect $b_o^{(t)}$ in (4.2) are updated in corresponding ways, based on (A2.3) and (A2.4). Thus,

$$b_o^{(t+1)} = b_o^{(t)} + b^{(t)} + [\lambda(1 - \frac{13}{24}\Lambda) + \Lambda]\ a_{t+1}, \qquad \text{(A2.19)}$$

since $(\Sigma m)/144 = 13/24$ and $\Sigma\delta_{12m} = 1$, and

$$b_m^{(t+1)} = b_{m+1}^{(t)} + [\ \frac{13 - 2m}{24}\lambda\Lambda - (1 - \delta_{12m})\Lambda]\ a_{t+1} \qquad \text{(A2.20)}$$

by subtracting (A2.19) from (A2.18). For $\lambda = .6$, $\Lambda = .4$

$$b_o^{(t+1)} = b_o^{(t)} + b^{(t)} + .87\ a_{t+1}$$

$$b_m^{(t+1)} = b_{m+1}^{(t)} + \begin{cases} [.01\ (13 - 2m) - .4]\ a_{t+1}, & m \neq 12 \\ -.11\ a_{t+1}, & m = 12 \end{cases}$$

Table A2.1 gives the updating coefficients for b_{om} and b_m for this fitted airline model.

Table A2.1

Coefficient of a_{t+1} in Updating Formulae for Fitted Airline Model

m	1	2	3	4	5	6	7	8	9	10	11	12	*
$b_{om}^{(t+1)}$	.58	.56	.54	.52	.50	.48	.46	.44	.42	.40	.38	.76	.02[a/]
$b_{m}^{(t+1)}$	-.29	-.31	-.33	-.35	-.37	-.39	-.41	-.43	-.45	-.47	-.49	-.11	.87[b/]

[a/] Slope $b^{(t+1)}$

[b/] Intercept $b_o^{(t+1)}$

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The importance of appropriate stochastic models in choosing efficient methods of statistical analysis is discussed. The fitting to data of Seasonal Autoregressive Moving Average models is described and it is shown how trend may be estimated in an appropriate class of models of this kind. The procedure is illustrated for a model fitted to a money supply series published by the Federal Reserve Board. Error limits are calculated. In a series of Appendices the properties of the adaptive coefficients which determine the trend estimates are derived.

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